

# Number and Algebra

## Chapter 5: Percentages

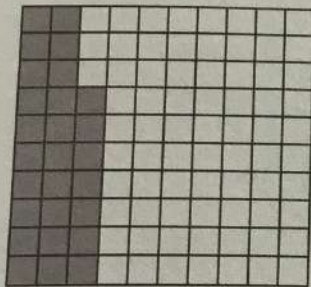
### Percentages and Fractions

A **percentage** is equivalent to a fraction whose denominator is 100.

The symbol % is used to mean **per cent** (or out of 100).

#### Example A

1. In the diagram below 27% (27 parts out of 100) of the squares are shaded.



2.  $5\% = \frac{5}{100}$  [since % means 'out of 100']  
 $= \frac{1}{20}$  [simplifying]

Fractions over 100 can be changed directly to percentages.

If a fraction is not over 100, it may convert easily to an equivalent fraction over 100.

A general method for converting a fraction to a percentage is to multiply the fraction by 100%.

#### Example B

1.  $\frac{29}{100} = 29\%$  [converts directly to %]
2.  $\frac{23}{50} = \frac{46}{100}$  [doubling both numbers]  
 $= 46\%$
3.  $\frac{1}{8} = \frac{1}{8} \times 100\%$  [multiply by 100 to get %]  
 $= \frac{100}{8}\%$  [ $\frac{1}{8} \times \frac{100}{1} = \frac{100}{8}$ ]  
 $= 12\frac{1}{2}\%$  [simplifying]

### Exercise A: Percentages and Fractions

1. Convert the following percentages to fractions in simplest form.

- |        |        |
|--------|--------|
| a. 37% | b. 89% |
| _____  | _____  |
| c. 15% | d. 90% |
| _____  | _____  |
| e. 2%  | f. 75% |
| _____  | _____  |

2. Convert to percentages

- |   |   |
|---|---|
| a. $\frac{45}{100} = \underline{\hspace{2cm}}\%$  | b. $\frac{3}{100} = \underline{\hspace{2cm}}\%$ |
| c. $\frac{129}{100} = \underline{\hspace{2cm}}\%$ | d. $\frac{4}{10} = \underline{\hspace{2cm}}\%$  |
| e. $\frac{35}{10} = \underline{\hspace{2cm}}\%$   | f. $1 = \underline{\hspace{2cm}}\%$             |

3. Use equivalent fractions over 100 to convert the following fractions to percentages.

- |   |
|---|
| a. $\frac{12}{25} = \frac{\square}{100} = \underline{\hspace{2cm}}\%$   |
| b. $\frac{7}{25} = \frac{\square}{100} = \underline{\hspace{2cm}}\%$    |
| c. $\frac{4}{5} = \frac{\square}{100} = \underline{\hspace{2cm}}\%$     |
| d. $\frac{27}{50} = \frac{\square}{100} = \underline{\hspace{2cm}}\%$   |
| e. $\frac{17}{20} = \frac{\square}{100} = \underline{\hspace{2cm}}\%$   |
| f. $\frac{1}{4} = \frac{\square}{100} = \underline{\hspace{2cm}}\%$     |
| g. $\frac{112}{200} = \frac{\square}{100} = \underline{\hspace{2cm}}\%$ |

## Percentages and Decimals

Whole number percentages are decimal fractions, which convert easily to decimals.

### Example C

- $37\% = \frac{37}{100}$  [by definition of %]  
 $= 0.37$  [37 hundredths]
- $5\% = \frac{5}{100} = 0.05$
- $450\% = \frac{450}{100} = 4.50$  or  $4.5$

In reverse, decimals with one or two decimal places convert easily to percentages.

### Example D

- $0.29 = \frac{29}{100} = 29\%$
- $0.08 = \frac{8}{100} = 8\%$
- $1.2 = 120\%$  [ $1.2 = \frac{120}{100}$ ]

Another strategy for converting percentages to decimals uses division by 100.

### Example E

- $5\% = 5 \div 100 = 0.05$  [dividing by 100 and using zero as a place holder]
- $2\frac{1}{2}\% = 2.5 \div 100 = 0.025$  [converting  $2\frac{1}{2}$  to a decimal]

To convert a decimal to a percentage, reverse the process and multiply by 100 (move decimal point 2 places right).

### Example F

Convert the following decimals to percentages.

- 0.6
- 1.09
- 0.075

#### Solution

- $0.6 = 0.6 \times 100\%$  [multiply by 100 and zero place holder]  
 $= 60\%$
- $1.09 = 1.09 \times 100\%$  [decimal point moves two places right]  
 $= 109\%$
- $0.075 = 0.075 \times 100\%$   
 $= 7.5\%$

## Exercise B: Percentages and Decimals

1. Convert the following percentages to decimals.

- |  |  |
|--|--|
| a. $19\% = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}$  | b. $25\% = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}$  |
| c. $90\% = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}$  | d. $7\% = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}$   |
| e. $160\% = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}$ | f. $200\% = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}$ |

2. Convert the following decimals to percentages:

- |   |   |
|---|---|
| a. $0.76 = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}\%$ | b. $0.4 = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}\%$  |
| c. $0.08 = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}\%$ | d. $0.35 = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}\%$ |

- |  |   |
|--|---|
| e. $1.4 = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}\%$ | f. $1.25 = \frac{\square}{100}$<br>$= \underline{\hspace{2cm}}\%$ |
|--|---|

3. Convert to decimals:

- |   |   |
|---|---|
| a. $2.5\% = 2.5 \div 100$<br>$= \underline{\hspace{2cm}}$                           | b. $175\% = \underline{\hspace{2cm}}$<br>$= \underline{\hspace{2cm}}$ |
| c. $8.5\% = \underline{\hspace{2cm}} \div 100$<br>$= \underline{\hspace{2cm}}$      | d. $0.3\% = \underline{\hspace{2cm}}$<br>$= \underline{\hspace{2cm}}$ |
| e. $64\frac{1}{2}\% = \underline{\hspace{2cm}} \div 100 = \underline{\hspace{2cm}}$ |   |
| f. $\frac{3}{10}\% = \underline{\hspace{2cm}} \div 100 = \underline{\hspace{2cm}}$  |   |

4. Use multiplication by 100 to convert the following decimals to percentages.

- |   |  |
|---|--|
| a. $0.4 = \underline{\hspace{2cm}}\%$   | b. $0.27 = \underline{\hspace{2cm}}\%$ |
| c. $1.4 = \underline{\hspace{2cm}}\%$   | d. $0.04 = \underline{\hspace{2cm}}\%$ |
| e. $0.325 = \underline{\hspace{2cm}}\%$ |  |
| f. $0.001 = \underline{\hspace{2cm}}\%$ |  |

5. Put in order of size from smallest to largest:  
 0.5%, 0.05, 5.5%,  $\frac{1}{500}$

## Percentages and Common Fractions

The percentage form of everyday fractions such as thirds and eighths should be known.

### Example G

1.  $\frac{3}{8} = \frac{375}{1000}$  [equivalent fraction over 1 000]  
 $= 0.375$   
 $= 37.5\%$  [ $\times 100$  to convert to %]  
 $= 37\frac{1}{2}\%$

2.  $33\frac{1}{3}\% = \frac{100}{3}\%$  [converting to improper fraction]  
 $= \frac{100}{3} \div 100$  [% means  $\div 100$ ]  
 $= \frac{1}{3}$  [100 thirds  $\div 100 = \frac{1}{3}$ ]

The following table gives the simplified fractions for many commonly used percentages.

Percentage to Fraction Conversions			
$5\% = \frac{1}{20}$	$10\% = \frac{1}{10}$	$15\% = \frac{3}{20}$	etc
$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$60\% = \frac{3}{5}$	$80\% = \frac{4}{5}$
$25\% = \frac{1}{4}$	$50\% = \frac{1}{2}$	$75\% = \frac{3}{4}$	$100\% = 1$
$12\frac{1}{2}\% = \frac{1}{8}$	$37\frac{1}{2}\% = \frac{3}{8}$	$62\frac{1}{2}\% = \frac{5}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$
$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$	$8\frac{1}{3}\% = \frac{1}{12}$	etc

This table should be memorised.

Note: A percentage such as  $112\frac{1}{2}\%$  is  
 $100\% + 12\frac{1}{2}\% = 1 + \frac{1}{8}$  or  $1\frac{1}{8}$ .

## Exercise C: Percentages and Common Fractions

1. Convert the following percentages to fractions (from memory).

a.  $50\% =$  \_\_\_\_\_      b.  $12\frac{1}{2}\% =$  \_\_\_\_\_  
 c.  $80\% =$  \_\_\_\_\_      d.  $8\frac{1}{3}\% =$  \_\_\_\_\_  
 e.  $75\% =$  \_\_\_\_\_      f.  $33\frac{1}{3}\% =$  \_\_\_\_\_  
 g.  $66\frac{2}{3}\% =$  \_\_\_\_\_      h.  $5\% =$  \_\_\_\_\_

2. Convert the following percentages to mixed numbers in simplest form.

a.  $105\% =$  \_\_\_\_\_      b.  $150\% =$  \_\_\_\_\_  
 c.  $225\% =$  \_\_\_\_\_      d.  $137\frac{1}{2}\% =$  \_\_\_\_\_  
 e.  $166\frac{2}{3}\% =$  \_\_\_\_\_      f.  $220\% =$  \_\_\_\_\_  
 g.  $310\% =$  \_\_\_\_\_      h.  $116\frac{2}{3}\% =$  \_\_\_\_\_

3. Fill in the table with the missing quantities so that each row has equivalent amounts.

	Percentage	Fraction	Decima
a.		$\frac{3}{4}$	
b.	35%		
c.			0.3
d.		$\frac{6}{25}$	
e.	12.5%		
f.			0.

4. Convert to percentages.

a.  $1\frac{1}{10} =$  \_\_\_\_\_%      b.  $2\frac{3}{4} =$  \_\_\_\_\_%  
 c.  $1\frac{5}{8} =$  \_\_\_\_\_%      d.  $\frac{25}{12} =$  \_\_\_\_\_%  
 e.  $2 =$  \_\_\_\_\_%      f.  $\frac{5}{2} =$  \_\_\_\_\_%

5. Amy scored  $\frac{21}{25}$  (21 marks out of 25) in her English test,  $\frac{7}{8}$  in her French test and  $\frac{17}{20}$  in her Maths test.

- a. Convert these marks to percentages.  
 \_\_\_\_\_  
 \_\_\_\_\_
- b. Which was Amy's best score? \_\_\_\_\_

6. Ben scored  $\frac{5\frac{1}{2}}{10}$  for his essay. What is this as a percentage?  
 \_\_\_\_\_  
 \_\_\_\_\_

7. Complete the magic square with the missing percentages, so that each row, column and diagonal has the same sum.

4%		
	0.05	7%
		$\frac{3}{50}$

8. Show how Simon can use the fact that  $8\frac{1}{3}\% = \frac{1}{12}$  to find the percentage for

- a.  $\frac{5}{12}$  \_\_\_\_\_  
 b.  $\frac{5}{6}$  \_\_\_\_\_  
 c.  $\frac{1}{4}$  \_\_\_\_\_

### Expressing one Quantity as a Percentage of Another

To express a quantity as a percentage of a second quantity:

- Put the first quantity over the second quantity (this expresses the first quantity as a fraction of the second quantity).
- Change the fraction to a percentage.

The quantities must be in the same unit.

#### Example H

Express \$45 as a percentage of \$60.

#### Solution

The fraction is  $\frac{45}{60}$  which simplifies to  $\frac{3}{4}$ .

The percentage is 75%.

Profit is often worked out as a percentage of the cost price.

#### Example I

Harriet owns a pet shop. She buys a dog collar for \$20 and sells it for \$27. What percentage profit has she made?

#### Solution

Profit is  $27 - 20 = \$7$

Fraction profit =  $\frac{7}{20}$

=  $\frac{35}{100}$

[putting profit over cost price]

[multiplying both numbers by 5]

Percentage profit = 35%

These strategies can be used in reverse to solve equations of the form  $\square\%$  of 48 = 36.

#### Example J

What percentage of 48 is 36? [ $\square\%$  of 48 = 36]

First find the fraction:  $\frac{36}{48} = \frac{3}{4}$  [simplifying]

Then convert to a percentage:  $\square = 75\%$

### Exercise D: Expressing one Quantity as a Percentage of Another

1. What percentage is the first quantity of the second?

a. 14, 35

b. 25, 40

\_\_\_\_\_

\_\_\_\_\_

c. 17.5, 35

d. 72, 60

\_\_\_\_\_

\_\_\_\_\_

e. 2.4, 3.6

f.  $1\frac{1}{2}$ ,  $2\frac{1}{2}$

\_\_\_\_\_

\_\_\_\_\_

2. Solve the equations to find the missing quantity.

- a. \_\_\_\_\_ % of 25 = 5
- b. \_\_\_\_\_ % of 36 = 9
- c. \_\_\_\_\_ % of 60 = 45
- d. \_\_\_\_\_ % of 12 = 36
- e. \_\_\_\_\_ % of 32 = 12
- f. \_\_\_\_\_ % of 6.5 = 3.9

3. Sophie used 2.5 mL from her 50 mL bottle of perfume.

- a. What percentage did she use? \_\_\_\_\_
- b. What percentage was left? \_\_\_\_\_

4. 17 people out of 25 interviewed agreed with a proposal to install a set of traffic lights at a busy intersection.

- a. What percentage of people agreed with the proposal?  
\_\_\_\_\_
- b. What percentage of people disagreed with the proposal?  
\_\_\_\_\_

5. In Georgie's netball trials, 7 out of her 10 attempts at goal were successful. Her friend, Hannah, was successful in 3 out of her 4 attempts at goal.

- a. What percentage of shots did Georgie get into the goal?  
\_\_\_\_\_
- b. What percentage of shots did Hannah get in goal?  
\_\_\_\_\_
- c. Which girl was more successful?  
\_\_\_\_\_



6. Express the first quantity as a percentage of the second quantity.

First quantity	Second quantity	First quantity as % of second
a. \$15	\$25	
b. 160 km	200 km	
c. 25 mL	500 mL	
d. 18 kg	12 kg	
e. 72 cm	108 cm	
f. 50 cents	\$5	

7. Find the percentage profit for the following.

	Cost price	Selling price	Profit	% profit
a.	\$40	\$50		
b.	\$125	\$175		
c.	\$400	\$550		
d.		\$75	\$25	
e.		\$110	\$30	
f.		\$720	\$80	

8. Barbara buys a skirt for \$50 to sell in her shop. The skirt does not sell quickly, so she reduces the price to \$40 to sell it below cost price. What percentage loss does Barbara make on the skirt?  
\_\_\_\_\_  
\_\_\_\_\_

9. Ben's hourly rate goes up from \$12 per hour to \$13.80 per hour. What percentage pay rise does he get?  
\_\_\_\_\_  
\_\_\_\_\_

10. Petra has 25 maths problems to do for homework. She finds she can do most of them easily, but there are three she cannot do. What percentage of the problems can she do?  
\_\_\_\_\_  
\_\_\_\_\_

## Finding a Percentage of a Quantity

To find a straightforward percentage of a quantity, first convert the percentage to a fraction.

### Example K

Find

1. 20% of 45      2.  $66\frac{2}{3}\%$  of 72.

#### Solution

1.  $20\% = \frac{1}{5}$  [from table of common conversions]

$$\frac{1}{5} \text{ of } 45 = 45 \div 5 = 9$$

So 20% of 45 = 9

2. Since  $66\frac{2}{3}\% = \frac{2}{3}$ , find  $\frac{2}{3}$  of 72

$$\frac{1}{3} \text{ of } 72 = 72 \div 3 = 24$$

$$\frac{2}{3} \text{ of } 72 = 24 \times 2 = 48$$

Alternatively, decimal multiplication can be used.

### Example L

$$\begin{aligned} 60\% \text{ of } 30 &= 0.6 \times 30 \\ &= 18.0 \\ &= 18 \end{aligned} \quad [30 \times 6 = 180 \text{ tenths}]$$

Note: Calculators can be used to do decimal multiplication quickly and accurately.

Other multiplication strategies can be used, such as doubling and halving.

### Example M

$$\begin{aligned} 35\% \text{ of } 200 &= 70\% \text{ of } 100 \quad [\text{doubling and halving}] \\ &= 0.70 \times 100 \\ &= 70 \end{aligned}$$

A more general rule is:

To find a percentage of a quantity, multiply the percentage by the quantity.

### Example N

$$\begin{aligned} 8\% \text{ of } \frac{1}{4} &= \frac{8}{100} \times \frac{1}{4} \quad [\text{'of' means multiply}] \\ &= \frac{8}{400} \quad [\text{multiplying fractions}] \\ &= \frac{1}{50} \quad [\text{dividing by 8}] \end{aligned}$$

## Exercise E: Finding a Percentage of a Quantity

1. Use the percentage-to-fraction conversion table to find:

a. 10% of 850      b.  $33\frac{1}{3}\%$  of 3.6

\_\_\_\_\_

c. 25% of 120      d. 20% of 0.75

\_\_\_\_\_

e.  $12\frac{1}{2}\%$  of 96      f.  $8\frac{1}{3}\%$  of 0.48

\_\_\_\_\_

2. Use the percentage to fraction conversion table to find:

a. 80% of 15      b.  $62\frac{1}{2}\%$  of 7.2

\_\_\_\_\_

c. 125% of 6      d.  $87\frac{1}{2}\%$  of 0.16

\_\_\_\_\_

e.  $37\frac{1}{2}\%$  of 40      f.  $66\frac{2}{3}\%$  of 21

\_\_\_\_\_

g. 120% of 80      h. 175% of 7

\_\_\_\_\_

3. Convert to decimal multiplications and evaluate without using a calculator.

a.  $12\%$  of  $50 = \underline{\hspace{1cm}} \times 50$   
 $= \underline{\hspace{1cm}} \times 10$   
 $= \underline{\hspace{1cm}}$

b.  $45\%$  of  $200 = \underline{\hspace{1cm}} \times 200$   
 $= \underline{\hspace{1cm}} \times 100$   
 $= \underline{\hspace{1cm}}$

c.  $15\%$  of  $60 = \underline{\hspace{1cm}} \times 60$   
 $= \underline{\hspace{1cm}} \times 10$   
 $= \underline{\hspace{1cm}}$

d.  $80\%$  of  $90 = \underline{\hspace{1cm}} \times 90$   
 $= \underline{\hspace{1cm}} \times 10$   
 $= \underline{\hspace{1cm}}$

e.  $62\%$  of  $40 = \underline{\hspace{1cm}} \times 40$   
 $= \underline{\hspace{1cm}} \times 10$   
 $= \underline{\hspace{1cm}}$

f.  $120\%$  of  $50 = \underline{\hspace{1cm}} \times 50$   
 $= \underline{\hspace{1cm}} \times 10$   
 $= \underline{\hspace{1cm}}$

4. Use multiplication strategies to complete the equations.

a.  $25\%$  of  $148 = 100\%$  of  $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

b.  $40\%$  of  $32 = 10\%$  of  $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

c.  $150\%$  of  $4.2 = 50\%$  of  $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

d.  $33\frac{1}{3}\%$  of  $96 = 100\%$  of  $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

e.  $75\%$  of  $24 = 25\%$  of  $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

f.  $15\%$  of  $12 = 30\%$  of  $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

5. Use multiplication of fractions to find answers in simplest fraction form.

a.  $20\%$  of  $\frac{3}{8}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

b.  $66\frac{2}{3}\%$  of  $0.9$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

c.  $30\%$  of  $\frac{6}{5}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

d.  $75\%$  of  $\frac{5}{12}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

e.  $140\%$  of  $\frac{1}{2}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

f.  $16\frac{2}{3}\%$  of  $3\frac{1}{4}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

6. Belinda runs  $35\%$  of a  $600$  m race, then has to stop because of cramp in her leg. How far had she run before she got cramp?

\_\_\_\_\_

\_\_\_\_\_

7. A test is marked out of  $40$ . If Paul got  $70\%$  of the test right, how many marks did he score?

\_\_\_\_\_

\_\_\_\_\_

8. Harry and Piripi help the teacher move  $150$  kg of text books. If Piripi moves  $60\%$  of the books, what weight of books does Harry move?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



**Example Q**

$66\frac{2}{3}\%$  of a journey is 150 km.

How long is the journey?



**Solution**

$$66\frac{2}{3}\% = \frac{2}{3}$$

$$\text{Journey is } 150 \div \frac{2}{3} \times 3 = 75 \times 3$$

$$[150 \div 2 = 100 \div 2 + 50 \div 2]$$

$$= 225 \text{ km.}$$

Decimal strategies may also be used.

**Example R**

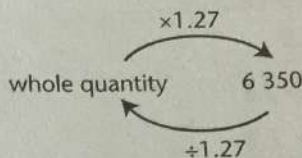
127% of a quantity is 6 350. Find the whole quantity.

**Solution**

$$127\% = 1.27$$

Multiplying the whole quantity by 1.27 gives 6 350.

In reverse, dividing 6 350 by 1.27 gives the whole quantity.



By calculator,  $6\,350 \div 1.27 = 5\,000$

**Exercise F: Finding the Whole Quantity if a Percentage is Known**

- A percentage of a quantity is given. Complete the working to find the whole quantity.
  - 5% is 25  
so 1% is \_\_\_\_\_  
and 100% is \_\_\_\_\_
  - 8% is 5.6  
so 1% is \_\_\_\_\_  
and 100% is \_\_\_\_\_
  - 3% is  $\frac{3}{5}$   
so 1% is \_\_\_\_\_  
and 100% is \_\_\_\_\_
  - 12% is  $\frac{24}{25}$   
so 1% is \_\_\_\_\_  
and 100% is \_\_\_\_\_
  - 110% is 44  
so 1% is \_\_\_\_\_  
and 100% is \_\_\_\_\_
  - 15% is 4.5  
so 1% is \_\_\_\_\_  
and 100% is \_\_\_\_\_
- Use fraction techniques to find the whole amount.
  - 20% of the whole amount is 3  
\_\_\_\_\_
  - 25% of the whole amount is 7  
\_\_\_\_\_
  - 50% of the whole amount is 3.7  
\_\_\_\_\_
  - $33\frac{1}{3}\%$  of the whole amount is  $\frac{3}{10}$   
\_\_\_\_\_
  - 10% of the whole amount is 0.6  
\_\_\_\_\_
  - $12\frac{1}{2}\%$  of the whole amount is  $\frac{1}{2}$   
\_\_\_\_\_
  - 5% of the whole amount is 12  
\_\_\_\_\_
- Use fraction techniques to find the whole amount.
  - 60% of the whole amount is 24  
\_\_\_\_\_
  - 90% of the whole amount is 27  
\_\_\_\_\_
  - $62\frac{1}{2}\%$  of the whole amount is 0.25  
\_\_\_\_\_
  - 150% of the whole amount is 2.1  
\_\_\_\_\_
  - 175% of the whole amount is 28  
\_\_\_\_\_
  - $66\frac{2}{3}\%$  of the whole amount is  $\frac{4}{5}$   
\_\_\_\_\_
  - 70% of the whole amount is 105  
\_\_\_\_\_



**Exercise G: Increasing or Decreasing by a Percentage**

1. Increase:

- a. 60 by 10%                      b. 240 by 25%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- c. 95 by 20%                      d. 8 by 50%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- e. 24 by  $12\frac{1}{2}\%$                       f. 66 by  $33\frac{1}{3}\%$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

2. Decrease:

- a. 120 by 5%                      b. 64 by 75%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- c. 10 by 30%                      d. 28 by 25%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- e. 400 by 80%                      f. 40 by 12%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

3. a. Increase 35 km by 4%                      b. Decrease 70 L by 15%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- c. Increase \$81 by  $66\frac{2}{3}\%$                       d. Decrease 20 kg by 20%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- e. Increase \$350 000 by 40%                      f. Decrease 150 cm by 1%

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

4. The price of milk goes up by 4% from \$4.20 per 2 litre bottle. Give the new price to the nearest ten cents.

\_\_\_\_\_

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5. A jacket is reduced in a sale by 35%. If the original price was \$240, what is the sale price?

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\_\_\_\_\_

6. A car dealer buys a car for a cost price of \$10 000. He adds 20% profit to the cost price to work out the selling price.

- a. What selling price does he put on the car?



The car does not sell, so the dealer puts the car on at a special price which reduces the selling price by 20%.

- b. What special price does the dealer put on the car?

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- c. Explain why the reduced price is not the same as the cost price.

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### Mixed Percentage Problems

Word problems may require a variety of skills. Answers should be expressed in words.

#### Example U

Rick earns \$12 per hour working at a petrol station. His friend, Tom, earns \$15 per hour working at the cinema.

- Express Rick's wages as a percentage of Tom's wages.
- Rick gets a pay rise which brings him up to the same hourly rate as Tom. What percentage pay rise did Rick get?
- Shortly afterwards, Tom also gets a \$3 per hour pay rise. Compare the percentage increase in Tom's pay with that of Rick's.

#### Solution

1. Fraction is  $\frac{12}{15}$  or  $\frac{4}{5}$  [put Rick's pay over Tom's pay and simplify]

$$\frac{4}{5} = 80\% \quad \text{[converting to a \%]}$$

Rick's hourly pay is 80% of Tom's hourly pay.

2. Pay increase is \$3 per hour.

Fraction is  $\frac{3}{12}$  or  $\frac{1}{4}$  [put increase over original amount and simplify]

$$\frac{1}{4} = 25\% \quad \text{[converting to a \%]}$$

Rick got a 25% pay increase.

3. Fraction is  $\frac{3}{15} = \frac{1}{5}$

Increase = 20% [converting to a \%]

Tom's percentage increase is less than Rick's, even though the dollar amount of the increase was the same for both Tom and Rick.

Note: Tom's percentage increase was smaller because his pay was greater, so the increase was a smaller fraction of his earnings.



Calculators have a percentage key. Learn how it works on your calculator.

### Exercise H: Mixed Percentage Problems

1. Jenny sells boxes of organic vegetables each week, at \$40 per box. Jenny works out that it costs her \$25 to fill and deliver each box.

- a. What percentage profit does Jenny make on each box?

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Jenny usually sells 120 boxes per week, but after an advertising campaign her sales go up by 30 boxes per week.

- b. What percentage increase is this?

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To attract even more customers, Jenny decides to reduce her prices by 10%.

- c. How much does each box of vegetables cost now?

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2. Ryan wants a Rangerider mountain bike which costs \$650 brand new. Ryan can't afford this, so he buys a second-hand bike of the same brand for \$245. He spends \$145 on new tyres and other parts and gets the bike looking as good as new.

- a. Express the total price of the second-hand bike as a percentage of the cost of the new bike.

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Ryan worked for 10 hours fixing up the bike. He values his time at \$13 per hour, and adds the cost of his time to the total cost of his bike.

- b. What percentage of the new price did Ryan's bike cost him now?

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3. Dotti buys fifteen tops for her boutique. She pays \$35 each for them and she marks them up by 40% to get the selling price.

- a. How much did the tops cost Dotti?

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- b. What is the selling price of each top?

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Dotti sells ten of the tops at the marked price, then puts the remaining five tops on sale at \$28 each.

- c. What is the percentage loss (on the cost price) she makes on each of the last five tops?

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- d. i. How much did Dotti sell the fifteen tops for?

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- ii. How much profit did Dotti make altogether on the fifteen tops?

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- e. What overall percentage profit did Dotti make on the fifteen tops?

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4. Maddy is training for a swimming event. She swims 1 500 m on the first day, then increases this by 10% on the second day.

- a. How far did Maddy swim on the second day?

\_\_\_\_\_

\_\_\_\_\_

The third day's distance is an increase of 10% on the second day.

- b. How far does Maddy swim on the third day?

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\_\_\_\_\_

- c. Express the distance swum on the first day as a percentage of the distance swum on the third day. Round your answer to the nearest whole number.

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- d. Maddy thinks the distance on the third day will be 20% more than the distance on the first day. Comment on Maddy's idea.

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5. Zac sends 480 texts in June. In July this number increases by  $33\frac{1}{3}\%$ .



- a. How many texts did Zac send in July?

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In August Zac had exams, so he sent 20% fewer texts than he did in July.

- b. How many texts did Zac send in August?

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- c. In September Zac went on holiday overseas where texts were more expensive to send, so Zac sent only 64 texts. What percentage decrease on the number of texts sent in August is this?

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6. There are 115 damaged apples in a shipment. If this is 9.2% of the shipment, how many apples were in the shipment?

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### Percentage Revision Puzzle

Solve the problems and match answers (no units required) to letters to read the saying below.

- C 45% as a simplified fraction
- E 15% of 80
- G Increase 2 m by 1%. Give answer in centimetres (1 metre = 100 cm).
- H 14 students out of 20 passed a test. What percentage failed?
- I Decrease 25 by 30%
- L The deposit on a \$2 400 fridge is 4.5%. How many dollars is that?
- M Convert  $12\frac{1}{2}\%$  to a fraction
- N 200% of \$8
- P 10% of a quantity is 6.3. What is the quantity?
- R  $\frac{7}{25}$  as a percentage = \_\_\_\_\_%
- S Hannah got 18 marks out of 40. What percentage is that?
- T A 10% reduction reduces a price by \$5. What was the previous price?
- V Kip sold 5 of his 8 bikes. What percentage did he sell?
- X Convert 0.05 to a percentage
- Y Reduce 48 L by 2.5%

12 46.8 12 45 45 12 12

12 62.5 12 28 46.8 50 30 17.5 16 202

12 5  $\frac{9}{20}$  12 63 50

50 30 12  $\frac{1}{-}$  45 12 108 62.5 12 45

### Sequences

A sequence is

#### Example

The first term is 100.

The first term is 100.

The first term is 100.

The first term is 100.

Many sequences have a pattern.

Many sequences have a pattern.

Many sequences have a pattern.

#### Example

Some sequences are

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